Stochastic Methods and Nonlinear PDEs

PROGRAMME

Wednesday 5th

9:00 - 11:00	*Registration and Welcome Coffee*
11:15 - 11:30	*Introduction to the workshop*
11:30 - 12:30	**Rainer Buckdahn (Université de Bretagne Occidentale, Brest)** Games without Isaacs Condition and associated Bellman-Isaacs equation
12:30 - 14:00	*Lunch buffet*
14:00 - 15:00	**Julio D. Rossi (Universidad de Alicante, Spain & Universidad de Buenos Aires, Argentina)** Tug-of-War games and the ∞-Laplacian with spatial dependence
15:00 - 16:00	**Mikko Parviainen (University of Jyväskylä)** Regularity for tug-of war with noise
16:00 - 16:30	*Coffee break*
16.30 - 17:30	**Jiang-Lun Wu (Swansea University)** A link of SDEs to nonlinear PDEs and optimal control for SDE associated with Lévy type operators

Thursday 6th

10:00 - 11:00	**Mauro Mariani (Université Aix-Marseille III)** Deviations for averaging stochastic reaction-diffusions equations
11:00 - 11:30	*Coffee break*
11:30 - 12:30	**Martin Hairer (University of Warwick)** Solving the KPZ equation
12:30 - 14:00	*Lunch*
14:00 - 15:00	**Elisabetta Carlini (Sapienza Università di Roma)** A Semi-Lagrangian scheme for the Mean Curvature equation
15:00 - 15:30	*Coffee break*
15:30 - 16:30	**Tim N. Phillips (Cardiff University)** Mathematical and computational models for Polymeric Liquids based on descriptions of the microstructure
19:00	*Social dinner*

Friday 7th

9:30 - 10:30	**Kenneth H. Karlsen (University of Oslo)** Stochastic conservation laws
10.:30 - 11:30	**Terry Lyons (University of Oxford)** Cubature
11:30 - 12:00	*Coffee break*
12:00 - 13:00	**Panagiotis E. Souganidis (University of Chicago)** Random homogenization for first and second order
13:00 - 14:30	*Lunch Buffet*

Rainer Buckdahn

Value in Mixed strategies for Zero-Sum Stochastic Differential Games without Isaacs Condition and associated Bellman-Isaacs equation

We consider 2-person zero-sum stochastic differential games with a non-linear pay-off functional defined through a backward stochastic differential equation. Our main objective is to study for such a game the problem of the existence of a value without Isaacs condition. Not surprisingly, this requires a suitable concept of mixed strategies which was not known in the context of stochastic differential games to our best knowledge. For this we consider non-anticipative strategies with a delay defined through a partition π of the time interval $[0, T]$. The underlying stochastic controls for the both players are randomised along π by a hazard which is independent of the governing Brownian motion, and knowing the information available at the left time point t_{j-1} of the subintervals generated by π, the controls of Player 1 and Player 2 are conditionally independent over $[t_{j-1}, t_j)$. It is shown that the associated lower and upper value functions W^π and U^π converge uniformly on compacts to a function V, the so-called value in mixed strategies, as the mesh of π tends to zero. This function V is characterised as the unique viscosity solution of the associated Hamilton-Jacobi-Bellman-Isaacs equation.

Joint work by Juan Li (Shandong University, Weihai,P.R.C.), Marc Quincampoix and Rainer Buckdahn (Université deBretagne Occidentale, Brest, France)
E-mail: Rainer.Buckdahn@univ-brest.fr

Elisabetta Carlini

A Semi-Lagrangian scheme for the Mean Curvature Equation

A Semi-Lagrangian scheme for the approximation of the Mean Curvature Motion is presented. The approximation is obtained coupling a stochastic method for the approximation of characteristics (to be understood in a generalized sense) with a local interpolation. The main features of the scheme are that it can handle degeneracies, it is explicit and it allows for large time steps. The scheme can be adapted to approximate different type of degenerate parabolic equation. An extension of the scheme to Area Preserving Mean Curvature Flow and to the Affine Morphological Scale Space (AMSS) model will also be discussed. Convergence can be proved for a modified scheme, via a generalized version of the Barles-Souganidis theorem. We will discuss several features of these schemes via some numerical simulations.

E-mail: Carlini@Mat.Uniroma1.it

Martin Hairer

Solving the KPZ equation

The KPZ equation was originally introduced in the eighties as a model of surface growth, but it was soon realised that its solution is a "universal" object describing the crossover between the Gaussian universality class and the KPZ universality class. The mathematical proof of its universality however is still an open problem, in particular because of the lack of a good approximation theory for the equation. Indeed, the only known way so far to mathematically interpret solutions to the KPZ equation is to reduce it to a linear stochastic PDE via a non-linear transformation called the Cole-Hopf transform. Unfortunately, the resulting linear equation does itself lack a good approximation theory and many microscopic models do not behave well under the Cole-Hopf transform. In this talk, we present a new notion of solution to the KPZ equation that bypasses the use of the Cole-Hopf transform. Our approach also allows to factorise the solution map into a "universal" (i.e. independent of initial condition) measurable map, composed with a solution map with good continuity properties. This lays the foundations for a robust approximation theory to the KPZ equation, which is needed to prove its universality. As a byproduct of the construction, we obtain very detailed regularity estimates on the solutions, as well as a new homogenisation result.

E-mail: M.Hairer@Warwick.ac.uk

Kenneth H. Karlsen

Stochastic conservation laws

We will first provide an overview of the existing stability and continuous dependence theory for stochastic conservation laws driven by Wiener processes. We will then present an Lp framework of stochastic entropy solutions for scalar conservation laws perturbed by a Poisson white noise term. The entropy formulation leads to the L1-contraction principle, which implies the stability and uniqueness of stochastic entropy solutions.

Based on joint works with Gui-Qiang Chen (Oxford) and Qian Ding (Evanston) and with Imran H. Biswas and Ananta K. Majee (Bangalore).

E-mail: kennethk@math.uio.no

Terry Lyons

Cubature

E-mail: Terry.Lyons@maths.ox.ac.uk

Mauro Mariani

Deviations for averaging stochastic reaction-diffusions equations

We discuss fluctuations and deviations for a system of coupled stochastic reaction-diffusion equations featuring self-averaging phenomena. The reversible case allows some explicit computations and will be discussed in detail.
E-mail: mariani@cmi.univ-mrs.fr

Mikko Parviainen

Regularity for tug-of war with noise

This talk will address a two-player zero-sum game called tug-of-war with noise. We describe regularity results for the value functions of the game, and in particular establish a local Lipschitz estimate. As an application, the result provides a new proof for local Lipschitz continuity and Harnacks inequality for p-harmonic functions. The proof is based on a careful choice of strategies and is thus quite different from the original techniques of De Giorgi, Moser, or Nash.
E-mail: mikko.j.parviainen@jyu.fi

Tim N. Phillips

Mathematical and Computational Models for Polymeric Liquids Based on Descriptions of the Microstructure

The significant improvements that have been made in the mathematical modelling of polymeric liquids over the last decade or so have necessitated a revolution in the development of associated computational techniques. This is because models derived from microscopic considerations, which utilize a coarse-grained description of polymer dynamics cannot be formulated in terms of closed form constitutive equations (PDEs). Instead, the polymeric contribution to the stress may be determined by means of stochastic simulations of the polymer dynamics. In this talk the micro-macro approach for predicting the flow of polymeric liquids will be described with emphasis on techniques for the stochastic element of the computation. Techniques for reducing the statistical error in a stochastic simulation without increasing the number of trajectories that need to be simulated will also be described.
E-mail: PhillipsTN@cardiff.ac.uk

Julio D.Rossi

Tug-of-War games and the ∞-Laplacian with spatial dependence

In this talk we look for PDEs that arise as limits of values of Tug-of-War games when the possible movements of the game are taken in a family of sets that are not necessarily euclidean balls. In this way we find existence of viscosity solutions to the Dirichlet problem for an equation of the form $-\langle D^2v \cdot J_x(Dv); J_x(Dv)\rangle(x) = 0$, that is, an infinity Laplacian with spatial dependence. Here $J_x(Dv(x))$ is a vector that depends on the the spatial location and the gradient of the solution.
E-mail: Julio.Rossi@ua.es

Panagiotis E. Souganidis

Random homogenization for first and second order

E-mail: souganidis@math.uchicago.edu

Jiang-Lun Wu

A link of SDEs to nonlinear PDEs and optimal control for SDE associated with Lévy type operators

This talk will address two (dierent) links between stochastic dierential equations and nonlinear PDEs. A new link of stochastic dierential equations to Burgers-KPZ type equations via Girsanov transformation will be established. As its application, a characterization of path independent property of action functionals for stochastic dynamical systems will be discussed. The second part of the talk is devoted to optimal control for stochastic dierential equations associated with Lvy generators.
E-mail: J.L.Wu@swansea.ac.uk